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Redefintions of Histories by Measurements - An Explanation of "Nonlocality" Observed in EPR-Bohm Experiments

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Abstract

It is proved in the frame of standard quantum mechanics that selection of different ensembles emerging from measurements of an observable leads to identification of corresponding reductions of the initial, premeasured state. This solves the problem of "nonlocality" observed in EPR-Bohm-type experiments.

The phenomenon of "nonlocality" is well-established in EPR-Bohm-type experiments (EPRB). For example, in entangled two-particle systems [1,2], a choice of measurement in one of the two causally separated channels influences results of measurements in another channel. The observed correlations are predicted by quantum mechanics (QM); but since measurement procedures are not included in its formalism, their mutual, long-distance influence remains inexplicable. This difficulty is cleared up in this work by analyzing logical statements about quantum states in the frame of the standard QM formalism, and by clarifying the meaning of measurement procedures. This approach permits us to find logical connections otherwise hidden. It is proved that in EPRB all such connections are *local.* In general, the choice of measurement in one of the channels locally defines the reduction of a *premeasured* state, which in turn defines connections between channels in the usual way.

Let $|k_l\rangle, l = 1, 2, \cdots$, be eigenstates of an observable(or of a number of commuting observables), K, considered here as discrete, with all k_l's different, and let $|\Psi\rangle$ be an arbitrary state of a physical system, $|\Psi\rangle = \sum_l a_l|k_l\rangle$. An exact measurement of K with result $K = k_i$ leads to the reduction $|\Psi\rangle \Rightarrow |k_i\rangle$. It is generally accepted, without proof, that reduced states describe only developments of physical systems *subsequent* to measurements. Below, in Theorem 3, we prove that under special conditions a reduction of a state may act in both directions of time, so one can replace premeasured state $|\Psi\rangle$ by its reduced part, using the information received from the measurement.

Lemma 1. The truth of a logical statement about a numerical value of a physical observable is itself an observable, and is represented in quantum mechanics by the pure state density matrix or by a sum of such matrices.

Proof. Consider the logical statement Λ_{k_l}: "The system is in state $|k_l\rangle$" or, in short, "$K = k_l$." Λ_{k_l} can be either "true" (the numerical value 1) or "false" (the value 0); these are two values of the classical logical *truth of statement* Λ_{k_l}. Statements corresponding to different k_j's (let us call them "elementary") are mutually exclusive. Λ_{k_l} is true if the measurement of K results in k_l, and false otherwise. Therefore, Λ_{k_l} is an observable and, like any other observable in QM, should be represented by a Hermitian operator, $\hat{\Lambda}_{k_l}$, the "truth operator of statement Λ_{k_l}," $\left(\hat{\Lambda}_{k_l}\right)^2 = \hat{\Lambda}_{k_l}$. $\hat{K}$ and $\hat{\Lambda}_{k_l}$ have a common set of eigenvectors, $|k_j\rangle, j = 1, 2, \cdots; \hat{\Lambda}_{k_l}|k_j\rangle = \delta_{lj}|k_j\rangle$. Here δ_{lj} is an eigenvalue: Λ_{k_l}=1 ("true") if $j = l$, and Λ_{k_l}=0 ("false") otherwise. We conclude that $\hat{\Lambda}_{k_l}$ must be identified with the density matrix of the pure state $|k_l\rangle : \hat{\Lambda}_{k_l} \equiv |k_l\rangle\langle k_l|$. Since our choice of K was arbitrary, a density matrix of any pure state $|\Psi\rangle$ defined in the Hilbert space of the physical system, $\hat{\Lambda}_\psi \equiv |\Psi\rangle\langle\Psi|, \hat{\Lambda}_\psi|\Psi\rangle = |\Psi\rangle$, and $\hat{\Lambda}_\psi|\overline{\Psi}\rangle$=0, where $|\overline{\Psi}\rangle$ is any state orthogonal to $|\Psi\rangle$, represents the logical statement, "The system is in state $|\Psi\rangle$." The truth of such a statement depends on whether the physical system is really in state $|\Psi\rangle$.

It is easy to show [3,4] that every nonelementary statement about numerical values of commuting physical observables is represented by a corresponding sum of density matrices. If there is a degeneracy such that the same state corresponds to $k_{l_1}, k_{l_2}, \cdots$, then the statement that the system is in this state is represented by $\hat{\Lambda}_{k_{l_1}} + \hat{\Lambda}_{k_{l_2}} + \cdots$ □

Theorem 1. On the existence of exact locations. If the state of the system, $|\Psi\rangle$, and its observable, K, are such that $|\Psi\rangle$ can be written as a superposition

$$|\Psi\rangle = \sum_{l_1}^{ln} c_l|k_l\rangle, \tag{1}$$

then the system is located at some point k_i of K-space, provided that $|k_i\rangle$ is represented in the superposition.

Proof. Let us find the operator of the following statement about the numerical value of K:

$$\Lambda(\Psi, K) : \Lambda_{k_{l_1}} \vee \Lambda_{k_{l_2}} \vee \cdots \vee \Lambda_{k_{l_n}} \equiv (\text{“}K = k_{l_1}\text{“}) \vee (\text{“}K = k_{l_2}\text{“}) \vee \cdots \vee (\text{“}K = k_{l_n}\text{“}), \quad (2)$$

in which only those k_l's that are in the superposition are represented. $\Lambda(\Psi, K)$ is symmetric relative to permutations of its constituent mutually exclusive elementary statements. Using Lemma 1, the condition of mutual exclusiveness in the operator representation, $\hat{\Lambda}_{k_l}\hat{\Lambda}_{k_m} = \delta_{lm}\hat{\Lambda}_{k_l}$, and equation $\left[\hat{\Lambda}(\Psi, K)\right]^2 = \hat{\Lambda}(\Psi, K)$, we find

$$\hat{\Lambda}(\Psi, K) = \sum_{l_1}^{ln} \hat{\Lambda}_{k_l} \equiv |k_{l_1}\rangle\langle k_{l_1}| + |k_{l_2}\rangle\langle k_{l_2}| + ... + |k_{l_n}\rangle\langle k_{l_n}|. \quad (3)$$

From this

$$\hat{\Lambda}(\Psi, K)|\Psi\rangle = |\Psi\rangle, \quad (4)$$

i.e., when the system is in state $|\Psi\rangle, \Lambda(\Psi, K)$ is a true statement. Thus, according to the meaning of $\Lambda(\Psi, K)$, this system is located in K-space at one of the points enumerated in (2).

Corollary. The statement-disjunction, $\Lambda_{k_1} \vee \Lambda_{k_2} \vee \cdots$, containing *all* possible numerical values of K, is true in *every* state-superposition (1).

From Theorem 1 it follows, for example, that when a (nonrelativistic) system is in a state of a certain momentum, $|p\rangle$, it is also certain that this system is located *somewhere* in the coordinate q-space. Such a statement does not contradict QM, but where this location is remains uncertain. The next theorem shows why such a question has no logical meaning.

Theorem 2. If K and L are two noncommuting observables, and the state (1) of system, $n > 1$, is an eigenstate of operator $L, L|\Psi\rangle = l|\Psi\rangle$, then there does not exist a logical statement, either true or false, about the exact location of this system in K-space.

We will omit the formal proof of this theorem. Nor will we discuss here the origins of noncommutativity. If noncommutativity is granted, then Theorem 2 provides the basis for quantum indeterminism [4]: in the general case, we cannot describe with certainty the future of the system undergoing a measurement. The following theorem states that this can be incorrect for the past.

Theorem 3. On the redefinition of history by measurements. Let K be an observable such that, in Heisenberg's representation, the commutator, $\hat{K}(t-\delta t)\hat{K}(t) - \hat{K}(t)\hat{K}(t-\delta t) \to 0$ when the time interval $\delta t \to 0$. (In particular, $\hat{K}$ may not depend on time at all.) Let the measuring procedure satisfy the condition of "ideal measurement" formulated below. And let a single measurement at time t_0, by definition, result in some $K{=}k$ only when statement $(\Lambda_k)_{t_0 msr}$ about that measured value is formulated, and the case separated from cases $k \neq k$; the corresponding postmeasured ensemble will be called "ensemble $K = k$." The following logical implication is true: If a measured state is a superposition, $|\Psi\rangle = \sum_l c_l |k_l\rangle$, and a single measurement results in $K = k_j$ at time t_0, then *after* this measurement, the statement: "*Before* t_0, the system belonging to the ensemble $K = k_j$ was in state $|k_j\rangle$" is true.

Proof. First we formulate a condition of an "ideal measurement," a postulate that is commonly assumed and does not contradict practice. We assume that an observable, K, is being measured. If the measured system is in state $|k_l\rangle, l = 1$, or 2, or 3,$\cdots$, then the result of the measurement is $K = k_l$. We will write this condition as the following logical implication (valid for every l):

$$(\Lambda_{k_l})_{t_0-\delta t} \to (\Lambda_{k_l})_{t_0 msr} \tag{5}$$

Statement (5) makes sense since its constituent statements, being formalized in QM, are assumed to commute, at least in the approximation $\delta t \to 0$; otherwise, according to Theorem 2, it would be meaningless. From (5) it follows that if the premeasured system is in one of the states $|k_j\rangle$ such that $j \neq l$, i.e., in any eigenstate of K but $|k_l\rangle$, then the result of the measurement of K is a $k_j, j \neq l$. This can be written as

$$\Big(\Lambda_{k_1} \vee \Lambda_{k_2} \vee \cdots \vee \Lambda_{k_{l-1}} \vee \Lambda_{k_{l+1}} \vee \cdots\Big)_{t_0-\delta t} \to \Big(\Lambda_{k_1} \vee \cdots \vee \Lambda_{k_{l-1}} \vee \Lambda_{k_{l+1}} \vee \cdots\Big)_{t_0 msr} \tag{6}$$

The premise is true if at least one of the elementary statements on the left side is true, and this is indeed the case; then according to (5), that very same elementary statement on the right side is true also, so the conclusion is also true. Now, according to the corollary of Theorem 1, the disjunction of *all* possible elementary statements $\Lambda_{k_l}, l = 1, 2, \cdots$, about numerical values of K is always a true statement (a tautology). Therefore, the disjunction from which only one elementary statement is excluded, as it is in (6), is a logical equivalent (denoted by $\sim$) of the logical negation of this statement:

$$\Lambda_{k_1} \vee \Lambda_{k_2} \vee \cdots \Lambda_{k_{l-1}} \vee \Lambda_{k_{l+1}} \vee \cdots \sim \bar{\Lambda}_{k_l} \tag{7}$$

Indeed, if one of the statements on the left side of the equivalence is true, then the left side is true and all other statements about numerical values of K, Λ_{k_l} included, are false; therefore, $\bar{\Lambda}_{k_l}$ on the right side is true. If none of the statements on the left side is true, then the left

side is false and Λ_{k_l} is true, as only possibility left to the physical system. Therefore, $\bar{\Lambda}_{k_l}$ is false, as is the left side. As a result, we can rewrite (6) as

$$\left(\bar{\Lambda}_{k_l}\right)_{t_0-\delta t} \to \left(\bar{\Lambda}_{k_l}\right)_{t_0msr} \tag{8}$$

And finally, as logically follows from (8),

$$\left(\Lambda_{k_l}\right)_{t_0msr} \to \left(\Lambda_{k_l}\right)_{t_0-\delta t} \tag{9}$$

Implication (9) can be used to conclude whether the state of the system was really $|k_l\rangle$, only when it is certain that $\left(\Lambda_{k_l}\right)_{t_0msr}$ is true, that is, when the results of the measurement formulated as "$K = k_l$" have been selected. Absent this procedure, the detector is not being used as a measuring device but only as a target. In such a case, some quantum state emerges as a result of the interaction of the system with this target, so premise $\left(\Lambda_{k_l}\right)_{t_0msr}$ in (9) is not true.

For the ensemble selected as $K = k_l$, the a posteriori conclusion from (9) and from the results of measurement is that the state of the system before the measurement was $|k_l\rangle$[5,6]. $\square$

Corollary. If an observable K satisfies the commutation conditions of Theorem 3, then when we select an ensemble corresponding to a result K=k of our measurement, we simultaneously select an ensemble corresponding to the value K=k of *premeasured* physical systems. (Note that the collapse of the premeasured state accompanying this selection is a purely informational effect.)

In EPRB [1,2,7-9] the observables are polarizations of particles, their operators do not depend on time, and δt may be finite. In such an experiment, let a pair of particles be prepared in an entangled state $|\Psi\rangle$ at time t=0 (for the center of their wave packet), and the time-size of the wave packet, $\triangle t \sim \hbar/\triangle E$, be much less than the flight time to either of two detectors. Let detector D1 measure observable K of particle 1 moving in channel 1, and L be an observable of particle 2 in channel 2 with its detector, D2. In the general case,

$$|\Psi\rangle = \sum_{ij} a_{ij}|k_i\rangle_1|l_j\rangle_2, \tag{10}$$

where 1(2) refers to particle 1(2), and k_i, l_j are eigenvalues of K,L. The phenomenon called "nonlocality" is the influence of random choices of noncommuting observables K, K', K'', $\cdots$, to measure, say, particle 1 in channel 1, on the outcomes of independent measurements of

particle 2 in the other channel. The significance of Theorem 3 is that it permits establishing deterministic logical connections, as in classical physics, between postmeasured states of particle 1 selected in channel 1, and premeasured states of particle 2 that are thereby selected also. Let observable K be measured at time t_0, and the postmeasured ensemble $K = k_n$ selected. Then it can be concluded that the premeasured state of particle 1 in ensemble $K = k_n$ is $|k_n\rangle_1$. But in common state (10), $|k_n\rangle_1$ is coupled one-to-one with $|l'_m\rangle_2 = A\sum_j a_{nj}|l_j\rangle_2$, where $1/|A|^2 = (w_{k_n})_1$ is the probability of a $(K = k_n)$ result in channel 1, and l'_m a value of an observable L'. Therefore, the state of coupled particle 2 is $|l'_m\rangle_2$. By selecting the one-particle, *post*measured ensemble $K = k_n$, then, we have also automatically selected the two-particle, *pre*measured ensemble described by the reduced state $|k_n\rangle_1|l'_m\rangle_2$. Thus, there are no nonlocal physical influences; "nonlocality" in the sense of one-to-one correspondence between selected states of the spatially separated particles, $k_n \leftrightarrow l'_m$, is a result of the common history of the two particles.

It is interesting that in EPRB we can even reconstruct the logical chain, with every infinitesimal link local, from the measurement in channel 1 to the state of particle 2 in channel 2. Since observables K and L in EPRB are constant before measurements, we conclude that at time $t_0 = N\delta t$ the value of K is the same as at time $t_0 - (N-1)\delta t, N = 1, 2, \cdots$. From Theorem 3 we know that at time $t_0 - \delta t, K = k_n$. Therefore, at time $t = 0$, when the pair of particles is born, the state of particle 1 in ensemble $K = k_n$ is $|k_n\rangle_1$. Therefore, the state of particle 2 at that moment is $|l'_m\rangle_2$. Applying similar logical steps to channel 2, we conclude that particle 2 conserves its state $L' = l'_m$ until some measurement in channel 2 is made either before or after t_0.

Let detector D2 now measure an observable, L. If L commutes with L', then according to (5) the result of this measurement is deterministic and equals $l = l'_m$. If L does not commute with L', the transition $|l'_m\rangle_2 \to |l_j\rangle_2$, where l_j results from the measurement of L, is unpredictable. Applying Theorem 3, now, not to the measurement of K but to the measurement of L by detector D2, and selecting postmeasured ensemble $L = l_j$, we will thereby automatically select a two-particle, premeasured ensemble different from the two-particle ensemble connected with ensemble $K = k_n$. Indeed, the result of measurement in channel 2, $L = l_j$, defines the state of particle 1 in channel 1, $|k'_p\rangle_1 = B\sum_i a_{ij}|k_i\rangle_1$, where k'_p is a value of an observable K'. From this we can conclude that the premeasured reduced state of the two-particle ensemble is $|k'_p\rangle_1|l_j\rangle_2$. Despite the differences between ensembles, however, probability $w\,(k_n, l_j) = \langle\Psi|\hat{\Lambda}_{k_n}\hat{\Lambda}_{l_j}|\Psi\rangle = |a_{nj}|^2$ for measurements in both channels does not depend on which of the two possible ensembles, $|k_n\rangle_1|l'_m\rangle_2$ or $|k'_p\rangle_1|l_j\rangle_2$, is chosen. Our a posteriori conclusion about the collapse of initial state $|\Psi\rangle$ therefore has an uncertainty caused by noncommutativity, and depends on the information we choose to be our premise

— either “$K = k_n$“ or “$L = l_j$“ (or both).

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